Fast Computation of Multiport Parameters of Multiconductor Coupled Microstrip Lines

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Abstract—**A fast approximate analysis method is presented for computing multiport parameters of uniform and nonuniform multiconductor coupled microstrip lines based on physical parameters considering dispersion and loss. A method for transient voltage analysis based on multiport S-parameters is also presented. The method is adequate for microwave and high-speed integrated circuit CAD software. Computed results for both S-parameters and transient voltages waveforms show good agreement with those by other methods or by measurement.**

I. Introduction

Multiconductor coupled microstrip lines are frequently employed in modeling Lange couplers, interdigitated filters and capacitors, meander lines, rectangular spiral inductors and parasitic couplings in hybrid and monolithic microwave integrated circuits as well as interconnects in high-speed integrated circuits.

For many years, a lot of research work has been done [1]–[7] in numerical analysis of multiconductor coupled microstrip lines in both frequency domain and time domain while Fourier transformation can be used to transfer responses between the two domains.

Frequency domain approaches can be divided into two types, quasistatic and full-wave. The quasistatic approach is valid only for lower frequencies. In the full-wave approach, dispersion is taken into account but the numerical solution of the Helmholtz eigenvalue equations is needed.

The data base technique of multidimensional lookup tables [8] has been employed to store data calculated by numerical full-wave approaches for applications in circuit CAD. Analytical formulas [9] are most suitable for CAD software. They may be obtained by curve fitting based on full-wave approaches. However, for microstrip lines, only analytical formulas considering dispersion and loss for single and symmetric 2-coupled lines are available in the published literature. D. G. Swanson [10] proposed a novel method of the generalized coupling model (GCM) in which existing formulas for single and symmetric 2-coupled microstrip lines were employed and good results were obtained.

In this paper a method for fast computation of general N-coupled microstrip lines is discussed and its simplicity and accuracy are shown.

II. Theory

The multiport parameters of multiconductor coupled microstrip lines (Fig. 1) with line length L shorter than a half-wavelength can be described by a $2n$-port Y-parameter matrix $[\mathbf{Y}]$.

The self-admittance item of each port and the transfer-admittance item of the two ports at opposite ends of each strip can be expressed as those of a single microstrip line with the same line width adding effects caused by all other strips (when $M = n-1$ in (2)) or a part of them (when $M < n-1$)

$$Y_{i,i} = Y_{i+n,i+n} = Y_{11}^{(i)} + \sum_{j=1,\, j\neq i}^{n} (Y_{11}^{(i,j)} - Y_{11}^{(i)})k_{ij} \qquad i = 1, 2, \cdots, n \tag{1}$$

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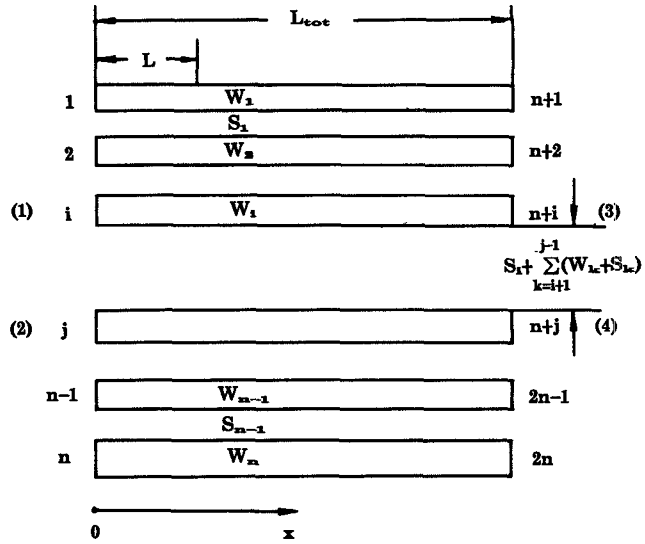

Fig. 1. N-coupled microstrip lines with $2n$ ports.

where

$$k_{ij} = \begin{cases} 1 & \text{for } |i-j| \le M \\ 0 & \text{for } |i-j| > M \end{cases} \tag{2}$$

$$Y_{i,i+n} = Y_{i+n,i} = Y_{12}^{(i)} + \sum_{j=1,\, j\neq i}^{n} (Y_{13}^{(i,j)} - Y_{12}^{(i)}) \qquad i = 1, 2, \cdots, n \tag{3}$$

Considering the effects on $Y_{i,i}$, $Y_{i+n,i+n}$, $Y_{i,i+n}$ and $Y_{i+n,i}$ caused by other strips in (1) and (3) is a major difference between this method and the method of GMC [10]. The transfer-admittance items of any two ports at strip-i and strip-j, respectively, can be simply expressed by those of 2-coupled microstrip lines consisting of strip-i and strip-j. They will be set to zero if $|i-j| > M$

$$Y_{i,j} = Y_{j,i} = Y_{i+n,j+n} = Y_{j+n,i+n} = Y_{12}^{(i,j)}k_{ij} \qquad i, j = 1, 2, \cdots, n \text{ and } i \neq j \tag{4}$$

$$Y_{i,j+n} = Y_{j+n,i} = Y_{14}^{(i,j)}k_{ij} \qquad i, j = 1, 2, \cdots, n \text{ and } i \neq j \tag{5}$$

$$Y_{j,i+n} = Y_{j+n,j} = Y_{23}^{(i,j)}k_{ij} \qquad i, j = 1, 2, \cdots, n \text{ and } i \neq j \tag{6}$$

In (1)–(6) $Y^{(i)}$ is the Y-parameter of a single microstrip line with line width W_i, and $Y^{(i,j)}$ is the four-port Y-parameter of 2-coupled microstrip lines consisting of line-i and line-j only.

For symmetric 2-coupled microstrip lines, the following formulas can be applied to obtain four-port Y-parameters:

$$Y_{11}^{(i)} = Y_0^{(i)} \coth(\gamma^{(i)} L) \tag{7}$$

$$Y_{12}^{(i)} = -Y_0^{(i)} \operatorname{cosech}(\gamma^{(i)} L) \tag{8}$$

$$Y_{11}^{(i,j)} = 0.5[Y_{0e}^{(i,j)} \coth(\gamma_e^{(i)} L) + Y_{0o}^{(i,j)} \coth(\gamma_o^{(i)} L)] \tag{9}$$

$$Y_{12}^{(i,j)} = 0.5[Y_{0e}^{(i,j)} \coth(\gamma_e^{(i)} L) - Y_{0o}^{(i,j)} \coth(\gamma_o^{(i)} L)] \tag{10}$$

$$Y_{13}^{(i,j)} = -0.5[Y_{0e}^{(i,j)} \operatorname{cosech}(\gamma_e^{(i)} L) + Y_{0o}^{(i,j)} \operatorname{cosech}(\gamma_o^{(i)} L)] \tag{11}$$

$$Y_{14}^{(i,j)} = Y_{23}^{(i,j)}$$

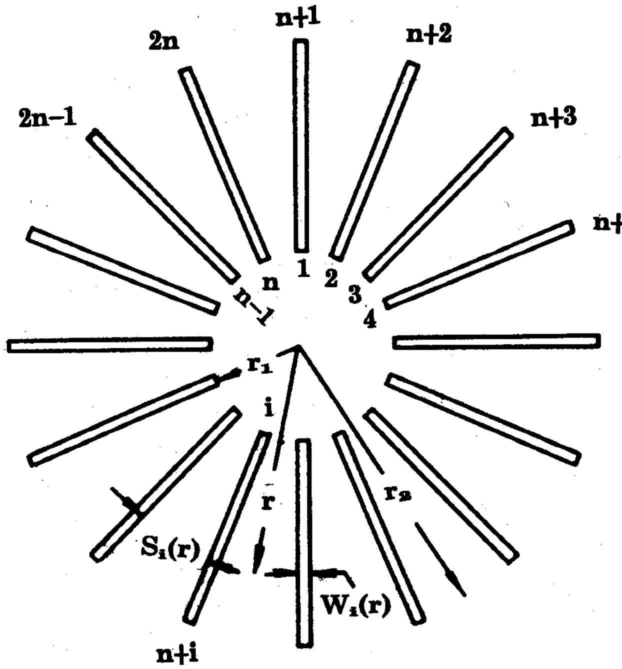

Fig. 2. N-closed-coupled microstrip lines.

$$= -0.5[Y_{0e}^{(i,j)} \operatorname{cosech}(\gamma_e^{(i)} L) - Y_{0o}^{(i,j)} \operatorname{cosech}(\gamma_o^{(i)} L)] \quad (12)$$

where $Y_0^{(i)}$ and $\gamma^{(i)}$ are the characteristic admittance and the propagation constant of the single microstrip line, respectively. $Y_{0e,o}^{(i,j)}$ and $\gamma_{e,o}^{(i,j)}$ are the even- and odd-mode characteristic admittances and propagation constants, respectively, of the 2-coupled microstrip lines. They can be calculated by analytical formulas in [9].

For asymmetric 2-coupled microstrip lines, we have to use the methods in [11] or [12] to get the 4-port Y-parameters, or the method in [6] to get 4-port S-parameters firstly and then transfer them to 4-port Y-parameters.

Then the $2n$-port Y-parameters obtained by (1)–(6) are transferred to $2n$-port S -parameters. There is a lost of accuracy at the half-wavelength frequency [10], so the line length L is restricted to shorter than a half-wavelength. For a longer line length L_{tot}, we can always divide the whole structure into several sections with the same length L which is shorter than a half-wavelength. After S -parameters of the small section have been calculated, the cascade formula can be used to get $[\mathbf{S}_{tot}]$ which is the S -parameter matrix of the whole structure with line length L_{tot} (Fig. 1).

Obviously this method can be extended to the analysis of nonuniform multiconductor coupled microstrip lines easily by cascading small sections with different physical dimensions.

This method is fast because analytical equations are used. The limitation of this method is that the effects of the strips between two nonadjacent strips are only taken into account in a network conception instead of a complete EM conception.

In packages and interconnects of high-speed integrated circuits the nonuniform multiconductor coupled microstrip lines are often adapted to form a closed loop in which line-1 is adjacent to line-n (Fig. 2). In that case we can use the above method by modifying (2) to

$$k_{ij} = \begin{cases} 1 & \text{for } |i-j| \geq M \text{ and } |i-j| < 1 + n/2, \\ & \text{or } ||i-j| - n| < M \text{ and } |i-j| \geq 1 + n/2. \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

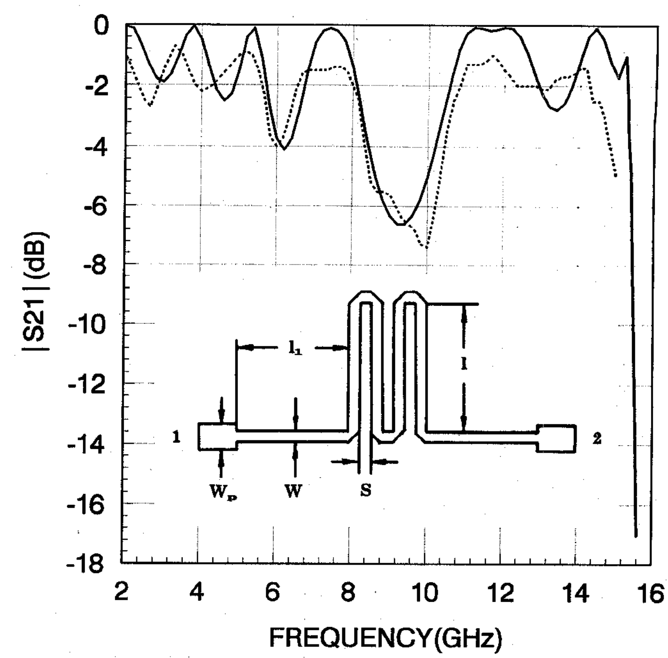

Fig. 3. Transmission properties of a two-port network with a meander line $W_p = 2.2$ mm, $W = 0.57$ mm, $S = 0.57$ mm, $l_1 = 8.05$ mm, $l = 9.77$ mm, $H = 0.75$ mm, $\epsilon_r = 2.86$, —— computed (lossless), - - - - measured [6].

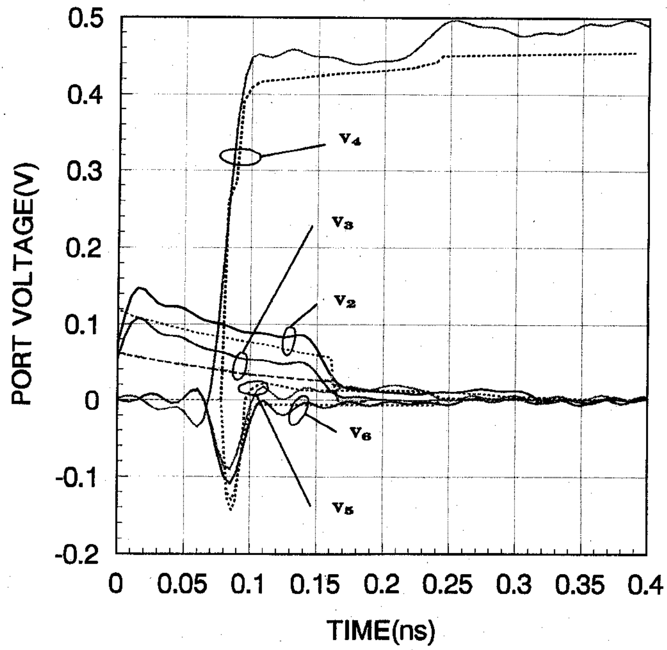

Fig. 4. Transient port voltages of nonuniform 3-coupled microstrip lines as response to a one volt step exitation with 100 ohm voltage source inner resistance at port 1 $W_1(x) = W_2(x) = W_3(x) = S_1(x) = S_2(x) = 10 + 40x/L_{tot}\,\mu$m, $L_{tot} = 10\,000\,\mu$m, $H = 100\,\mu$m, $\epsilon_r = 9.8$, this method , —— (lossless), - - - - [7].

M is usually set to 2 or 3. It should not be set too large, because only neighboring lines can be considered approximately parallel to each other.

In CAD of high-speed integrated circuits the transient voltage analysis is important to predict distortion and cross talk. By Fourier transform techniques, we can do transient analysis via S-parameter analysis. Now we consider a system consisting of a $2n$-port network terminated by voltage sources $v_{is}(t)(i = 1, 2, \cdots 2n)$ in series with

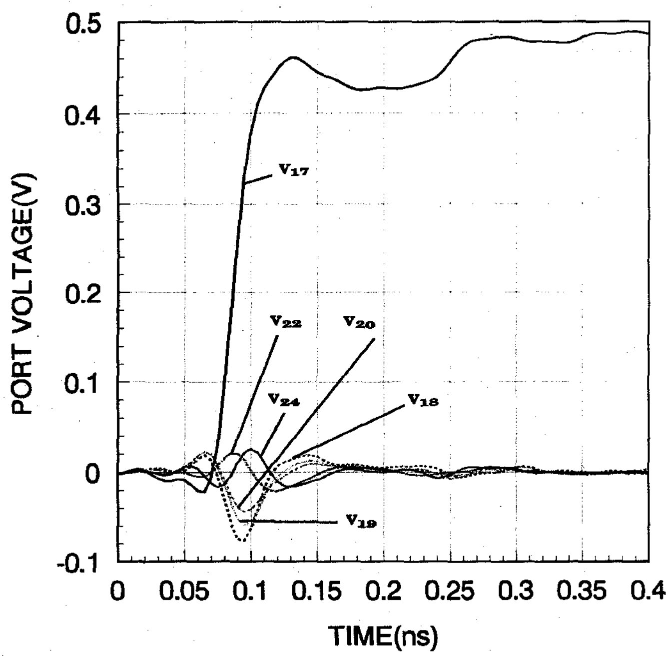

Fig. 5. Transient port voltages of nonuniform 16-closed-coupled microstrip lines as response to a one volt and 100 ohm inner resistance step exitation at port 1. $H = 0.5$ mm, $\epsilon_r = 9.8$, $W_i = 0.5$ mm ($i = 1, 2, \cdots, 16$), $Z_{Li} = 100$ ohm ($i = 1, 2, \cdots, 16$), $S_i(r) = 0.5 + 3.93(r - r_1)/(r_2 - r_1)$ mm ($i = 1, 2 \cdots, 16$), $r_1 = 2.546$ mm, $r_2 = 12.546$ mm, $M = 3$, $N = 16$.

impedance Z_{Li}. Firstly, $v_{is}(t)$ can be transferred to the frequency domain by

$$V_{is}(\omega) = \int_{-\infty}^{\infty} v_{is}(t)e^{-j\omega t}\, dt \tag{14}$$

and then port voltage $[\mathbf{V}(\omega)]$ can be calculated by

$$[\mathbf{V}(\omega)] = \operatorname{diag}[\sqrt{Z_{0i}}]([\mathbf{a}] + [\mathbf{b}]) \tag{15}$$

where Z_{0i} is the reference impedance of the ith port.

$$[\mathbf{a}] = ([\mathbf{1}] - [\Gamma][\mathbf{S}_{tot}])^{-1}[\mathbf{c}] \tag{16}$$

$$c_i = V_{is}(\omega)\sqrt{Z_{0i}}/(Z_{Li} + Z_{0i}) \quad (i = 1, 2, \cdots, 2n) \tag{17}$$

$$[\mathbf{\Gamma}] = \operatorname{diag}[(Z_{Li} - Z_{0i})/(Z_{Li} + Z_{0i})] \tag{18}$$

$$[\mathbf{b}] = [\mathbf{S}_{tot}][\mathbf{a}]. \tag{19}$$

Finally, $V_j(\omega)(j = 1, 2, \cdots 2n)$ can be transferred back to the time domain by

$$v_j(t) = \int_0^{\infty} |V_j(\omega)| \cos(\omega t - \varphi_j(\omega))\, d\omega/\pi \tag{20}$$

where $\varphi_j(\omega)$ is the phase angle of $V_j(\omega)$.

III. Results and Applications

In Fig. 3 the computed transmission coefficient of a two-port network containing a meander line is shown compared with measured data [6]. A meander line is often applied where the circuit layout is restricted to a small substrate. In the computation the microstrip discontinuities such as steps, truncated right angle bends are considered by a magnetic wall mode-matching method.

This method can also be applied to the signal integrity analysis in high-speed interconnect and packaging design. Fig. 4 is an example of nonuniform 3-coupled microstrip lines. Computed transient voltages as a response to a one volt step voltage source in series with a 100 ohm resistance at port 1 are compared with those by Orhanovic *et al.* [7]. In these examples, the truncation point of the integration in (20) is $f = 30$ GHz, and step voltage sources are replaced by rectangular voltage pulses with pulse-width of 1 ns. An example of nonuniform 16-closed-coupled microstrip lines is given in Fig. 5. Transient voltages at some ports as responses to a one volt step voltage source in series with a 100 ohm resistance are plotted.

This method is fast. For example, the computation time on a PC/386 is 0.82 second for a 10-coupled microstrip line structure and 0.13 second for a 5-coupled microstrip line structure, per frequency point respectively.

IV. Conclusion

A fast approximate method for analysis of multiport parameters of uniform and nonuniform multiconductor coupled microstrip lines has been presented. Dispersion and loss can be taken into account easily. A method for transient analysis based on multiport S-parameters has also been presented.

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